Pre-class Warm-up!!!

How would you do the following question?

Spring 2007 Midterm 3
2. Compute the integral ∬_S F · dS , where F(x,y,z) = (y-z, x-z, x-y) and S is the planar surface parametrized by Phi (u,v) = (u-v, u+v, u) for $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Orient the surface so the first component of the normal vector is positive.

√a. Evaluate directly

b. Go into polar, cylindrical or spherical coordinates.

c. Make a different change of variables.

d. Use Stokes' theorem ?

e. None of the above.

Review for exam 3

' We have covered $\nabla \cdot F = 0$ $\nabla \times F = 0$

8.3 Conservative vector field (criterion for F = grad f, also for F = curl G. All past exam questions ask to find f so that F = grad f and then evaluate a line integral $\int F \cdot dS$, which was done in section 7.2 and on Exam 2.)

$= f(c(b)) - f(c(a))$

There is no question like this on Exam 3.

6.1, 6.2 Change of variables in 2D and 3D. (Finding linear changes of variables, the change of variables formula, cylindrical and spherical coordinates) $\iint_D f(x,y)\,dx\,dy = \iint_{D^*} f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$

7.3 What is a parametrization of a surface? (Find tangent planes) Tangent vectors T_u, T_v

E Normal $T_u \times T_v$.

7.4 Surface area. $\iint_D \|T_u \times T_v\|\,du\,dv$.

7.5 Scalar surface integrals $\iint_D f \|T_u \times T_v\|\,du\,dv$

7.6 Vector surface integrals $\iint_S F \cdot d\underline{S} = \iint_D F \cdot (T_u \times T_v)\,du\,dv$.

8.2 Stokes' theorem With the boundary orientation

$$\int_{\partial S} F \cdot d\underline{s} = \iint_S \nabla \times F \cdot d\underline{S}$$

Things not on Exam 3

Find f so that F = grad f, then use this to compute a line integral.
This was material from section 7.2 and appeared in Exam 2.

Green's theorem. Ds

For cylindrical coordinates

$dx\,dy\,dz = r\,dr\,d\theta\,dz$

$dx\,dy\,dz = \text{——}\ d\rho\,d\theta\,d\phi$

formula given on the exam

The past Spring exams 2007 - 2011

All have a question about Stokes' theorem. Three do Stokes on a triangle, one does it on a hemisphere, and one on a disk parallel to the xy-plane.

Typically they have one or two questions about cylindrical or spherical coordinates.

They all have a change of variables question.

The mostly have a gradient vector field question. Not on Exam 3

Spring 2007 Midterm 3

2. Compute the integral ∬_S F · dS , where F(x,y,z) = (y-z, x-z, x-y) and S is the planar surface parametrized by Phi (u,v) = (u-v, u+v, u) for $0 \le u \le 1$ and $0 \le v \le 1$. Orient the surface so the first component of the normal vector is positive.

First approach: use Stokes' theorem (bad idea!)

Notice that $F = \nabla \times G$ for some G because $\nabla \cdot F = \left(\frac{\partial}{\partial x}(y-z), \frac{\partial}{\partial y}(x-z), \frac{\partial}{\partial z}(x-y)\right) = 0$

$G = \left(\frac{y^2-z^2}{2}, \quad , \quad \right)$.

$\iint_S F \cdot dS = \int_{\partial S} G \cdot d\underline{s}$

∂S = 4 edges of a parallelogram, giving 4 integrals

It is a lot of work.

2nd Approach: evaluate directly.

$T_u = (1,1,1) \quad T_v = (-1,1,0)$.

$T_u \times T_v = (-1,-1,2)$. Φ is not consistent with the orientation. To correct this use

$\Phi_1(u,v) = \Phi(v,u)$. Now $T_u \times T_v = (1,1,-2)$

Or use Φ and multiply by -1 at the end.

Using Φ_1 we get

$$\int_0^1\int_0^1 (u+v-u,\ u-v-u,\ u-v-(u+v)) \cdot (1,1,-2)\, du\, dv$$

$$= \int_0^1\int_0^1 (v-v+4v)\, du\, dv$$

$$= 2.$$

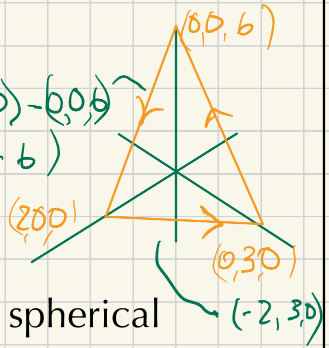
Spring 2010 Question 6.
Evaluate

$\int_C (x+y)dx + (2x-z)dy + (y+z)dz$

where C is the perimeter of the triangle connecting (2,0,0), (0,3,0) and (0,0,6) in that order.

How would you do it?

$(2,0,0)-(0,0,6) = (2,0,-6)$



a. Evaluate directly

b. Go into polar, cylindrical or spherical coordinates.

c. Make a different change of variables.

d. Use Stokes' theorem

e. None of the above.

Solution using Stokes. Let S be the triangle with these vertices, oriented by a normal vector pointing at us. Now $C = \partial S$

Stokes $\int_C F \cdot d\underline{s} = \iint_S \nabla \times F \cdot d\underline{S}$

$\nabla \times F = (1-(-1), 0-0, 2-1) = (2, 0, 1)$

We need $T_u \times T_v$. Parametrize S

$\Phi(u,v) = u(2,0,-6) + v(-2,3,0) + (2,0,0)$

$T_u = (2,0,-6) \quad T_v = (-2,3,0)$

$T_u \times T_v = (18, 12, 6)$. This has correct orientation. Let $\underline{n} = \frac{T_u \times T_v}{\|T_u \times T_v\|}$ be a unit normal.

$\iint_S \nabla \times F \cdot dS = \iint_D \nabla \times F \cdot \underline{n} \, \|T_u \times T_v\| \, du\,dv$

$= \frac{42}{\|T_u \times T_v\|} \iint_D \|T_u \times T_v\| \, du\,dv = \frac{42}{\|T_u \times T_v\|} \cdot \text{Area of } S$

$= \frac{42}{\|T_u \times T_v\|} \cdot \frac{1}{2} \cdot \|T_u \times T_v\| = 21$

Spring 2010 Question 1. Parametrize the surface $3(x^2+y^2) + 2z^2 = 2$ with $z \geq x^2+y^2$.

How would you do it?

a. Evaluate directly

b. Go into polar, cylindrical or spherical coordinates.

c. Make a different change of variables.

d. Use Stokes' theorem

e. None of the above.

Spring 2008 Question 4. Parametrize the ellipsoid $9x^2 + 4y^2 + z^2 = 36$. Include the correct bounds

Find an equation for the tangent plane when theta = phi = $\pi/4$.

Spring 2010 Question 5. Let Be be the region in the first quadrant bounded by the curves $xy = 1$, $xy = 3$, $x^2-y^2 = 1$ and $x^2-y^2 = 4$.

Evaluate $\iint_B (x^2+y^2)dx\, dy$

Using the change of variables $u = x^2 - y^2$, $v = xy$.